The halo model and cosmological power spectra

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Introduction

• Aim to understand the distribution of large-scale structure (matter) in the late (z < 2) Universe.

• Structure is traced out by galaxies, which we can see, but consists of a dark matter exoskeleton, often called the 'cosmic web'

• This structure can be simulated easily, and measured directly via weak gravitational lensing

• Different cosmological models predict very different patterns for the cosmic web, and can thus be distinguished using weak lensing.
The problem

- Structure in the late Universe is non-linear and complex
- Initial perturbations are linear and Gaussian
- Gravitational collapse makes things non-linear and non-Gaussian
- Late Universe must be modelled (e.g., weak lensing)
- Necessary to do this as a function of model (DE, MG, massive neutrinos)
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Linear perturbation theory

\[ 1 + \delta(x) = \frac{\rho(x)}{\bar{\rho}} \]

\[ \ddot{\delta} + 2H \dot{\delta} = 4\pi G \bar{\rho}_m \delta \]
Linear perturbation theory

\[ 1 + \delta(x) = \frac{\rho(x)}{\bar{\rho}} \]

\[ \ddot{\delta} + 2H \dot{\delta} = 4\pi G \bar{\rho}_m \delta \]

\[ \delta \propto a \]
The power spectrum of matter fluctuations

\[ \Delta^2(k, z) = 4\pi V \left( \frac{k}{2\pi} \right)^3 P(k, z) \]

\[ P(k, z) = \langle |\delta_k|^2 \rangle \]

\( k/(h \text{ Mpc}^{-1}) \)
N-body simulations

- Represent the smooth density field with 'N' particles
  - 1. Calculate the gravitational forces
  - 2. Move the particles
  - 3. Repeat
- Periodic boundaries
- Expanding box

\[
\ddot{\mathbf{r}} + 2H\dot{\mathbf{r}} = -\frac{1}{a^2} \nabla \Phi
\]
Simulation limitations

- Solve gravity only problem to high accuracy
- Expensive in terms of computing power
- It will never be possible to run accurate simulations for every model under consideration
- Investigating cosmological parameter space:
  - $\Lambda$CDM
  - Dark Energy (interacting?)
  - Massive neutrinos
  - Dark matter types
  - Modified gravity
  - Baryons?
The halo model

- Can be used to predict the clustering of matter (as well as of haloes or of galaxies) in the Universe
- Distinct from HALOFIT (Smith et al. 2003)

\[
\Delta^2(k) = \Delta^2_{2H} + \Delta^2_{1H}
\]

\[
\Delta^2_{1H}(k) = 4\pi \left( \frac{k}{2\pi} \right)^3 \frac{1}{\rho^2} \int_0^\infty M^2 W^2(k, M) f(M) \, dM
\]

large-scales $\sim$ linear

small scales - contribution from haloes
\[ \Delta_{1H}^2(k) = 4\pi \left( \frac{k}{2\pi} \right)^3 \frac{1}{\bar{\rho}^2} \int_0^\infty M^2 W^2(k, M) f(M) \, dM \]

\[
f(v) = A \left[ 1 + \frac{1}{(av^2)p} \right] e^{-av^2/2}
\]

\[
\nu = \frac{\delta_c}{\sigma(M)}
\]

\[
\rho(r) = \frac{\rho_N}{(r/r_s)(1+r/r_s)^2}
\]

\[
r_v = \left( \frac{3M}{4\pi\Delta \bar{\rho}} \right)^{1/3}
\]

\[
c(M, z) = A \frac{1+z_f}{1+z}
\]

**Ingredients**

- Halo mass function
- Peak to mass relation
- Halo density profiles
- Halo radius
- Halo concentration
\[ \Delta^2(k) = \Delta^2_{2H} + \Delta^2_{1H} \]

\[ \Delta^2_{2H} = \Delta^2_{\text{lin}} \]

\[ \Delta^2_{1H}(k) = 4\pi \left( \frac{k}{2\pi} \right)^3 \frac{1}{\bar{n}^2} \int_0^\infty M^2 W^2(k, M) f(M) \, dM \]
Duffy concentration-mass relation

\[ \Delta^2(k) \]

\[ \Delta^2_{HM}/\Delta^2_{emu} \]

Simulation

\[ z=0.0 \]
\[ z=1.0 \]

\[ k/(h \text{ Mpc}^{-1}) \]
Standard halo model

Problems

- Perturbation theory at large scales
- Transition region is problematic (voids, under-densities, NL bias)
- Filamentary structure missing
- Halo asphericity ignored
- Tidal alignment of haloes
- Assumes all objects virialized
- Halo substructure ignored
- Scatter in halo properties at fixed mass ignored
Method

• We want to remedy the inaccuracy of the halo model without breaking it

• Fixing all of the problems whilst being fully consistent would be hard

• Instead opt for the simpler goal of generating 'effective haloes', whose simple halo power spectrum accurately matches that of simulations
Variations in virial density
\[ \Delta_v = 200 \]

\[ \Delta_v = 400 \]

\[ \Delta_v = 100 \]

Variations in linear collapse density
\[ \delta_c = 1.686 \]

\[ \delta_c = 1.65 \]

\[ \delta_c = 1.7 \]
\[ \frac{\Delta^2(k)}{\Delta^2_{\text{emu}}} \]

Simulation

\[ z=0.0 \]

\[ z=1.0 \]

fitting

\[ \frac{\Delta^2_{\text{HM}}}{\Delta^2_{\text{emu}}} \]

\[ k/(h \text{ Mpc}^{-1}) \]
Cosmic Emu (simulations) fit and comparison (Heitmann 2014)
Baryonic feedback

\[ c(M, z) = A \frac{1 + z_f}{1 + z} \]

\[ \frac{\Delta^2}{\Delta^2_{\text{mid}}} \]

Parameter: 7

Lowest value

Highest value

A=10

A=1
Baryonic feedback

\[ \Delta^2(k) \]

\[ \frac{\Delta^2(k)}{\Delta^2_{\text{DMONLY}}} \]

\[ \Delta^2(k)_{\text{SIM}} \]

\[ \theta_\xi \] [arcminutes]

\[ \theta_\xi \] [degrees]

Simulation +
AGN
REF
DBLIM

Planck
CFHTLenS

-1 \times 10^{-6}
1 \times 10^{-6}
3 \times 10^{-6}

\[ k/(h \text{ Mpc}^{-1}) \]

\[ \theta [\text{arcminutes}] \]

\[ \theta [\text{degrees}] \]
From Joudaki et al. (CFHTLenS revisited;1601.05786)
Dark energy

\[ w = w_0 + (1 - a)w_a \]
Massive neutrinos

Simulation
Linear
Bird (2012)
Mead (2015)
This paper

$m_{\nu}=0.15 \text{ eV}; z=0$

$m_{\nu}=0.30 \text{ eV}; z=0$

$m_{\nu}=0.60 \text{ eV}; z=0$

$k / (\text{h Mpc}^{-1})$

$m_{\nu}=0.15 \text{ eV}; z=1$

$m_{\nu}=0.30 \text{ eV}; z=1$

$m_{\nu}=0.60 \text{ eV}; z=1$

$k / (\text{h Mpc}^{-1})$
Summary

• Matter power spectra accurate to 5% across a wide range of cosmological models (massive nu, dark energy)

• Publications
  • http://mnras.oxfordjournals.org/content/454/2/1958.full.pdf
  • http://mnras.oxfordjournals.org/content/459/2/1468.full.pdf

• Code available:
  • https://github.com/alexander-mead/HMcode

• Integrated into CAMB (halofit_ppf.f90 module)

• Relatively easy to add standard model extensions
Spherical collapse model

- 5% accuracy can be improved on if we work only with the power spectrum 'response'

- Consider models with fixed linear spectrum shape and amplitude, but that could differ via their growth history (e.g., dark energy)

- Look at differences in spherical-collapse model predictions.
Spherical-collapse model

\[ \ddot{\delta} + 2H \dot{\delta} - \frac{4}{3} \frac{\dot{\delta}^2}{1 + \delta} = 4\pi G \bar{\rho} \delta (1 + \delta) \]
Spherical-collapse model

\[ \ddot{\delta} + 2H \dot{\delta} - \frac{4}{3} \frac{\dot{\delta}^2}{1 + \delta} = 4\pi G \bar{\rho} \delta (1 + \delta) \]
Spherical-collapse model

\[ \ddot{\delta} + 2H\dot{\delta} - \frac{4}{3} \frac{\dot{\delta}^2}{1 + \delta} = 4\pi G \bar{\rho} \delta (1 + \delta) \]

\[ \ddot{\delta} + 2H\dot{\delta} = 4\pi G \bar{\rho} \delta \]
Table 1. Cosmological parameters of the simulations used in this paper. Dynamical dark energy is parameterised via \[ w(a) = w_0 + (1 - a)w_a \] and is taken to be spatially homogeneous and thus only affects the background expansion. All simulations use \( 512^3 \) particles in cubes of size \( L = 200h^{-1}\text{Mpc} \), and start from initial conditions with identical mode phases, but with initial amplitudes adjusted to ensure \( \sigma_8 = 0.8 \) at \( z = 0 \). The shape of the linear spectrum used to generate the initial conditions is identical in each case, and was generated using CAMB (Lewis et al. 2000) with cosmological parameters \( \Omega_m = 0.3, \Omega_w = 1 - \Omega_m, \Omega_b = 0.05, h = 0.7, n_s = 0.96 \) and \( w = -1 \). For each cosmology, I ran 3 different realisations of initial conditions. Also shown are the spherical-model parameters \( \delta_c \) and \( \Delta_v \) from a numerical calculation.

<table>
<thead>
<tr>
<th>Cosmology</th>
<th>( \Omega_m )</th>
<th>( \Omega_w )</th>
<th>( w_0 )</th>
<th>( w_a )</th>
<th>( \delta_c )</th>
<th>( \Delta_v )</th>
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<td>( \Lambda )CDM</td>
<td>0.3</td>
<td>0.7</td>
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<td>0</td>
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<td>310.1</td>
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<td>0.0</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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<td>290.1</td>
</tr>
</tbody>
</table>
Spherical collapse model

Real space

Redshift space

DE only; real space

DE only; redshift space

$k / (h \text{ Mpc}^{-1})$

$P(k_x) / P(k_{x,\Lambda CDM})$

$P_0(k_x) / P_0(k_{x,\Lambda CDM})$

$x = \text{Halo model}$

$x = \text{Takahashi (2012)}$

$x = \text{Simulation}$

$w_0 = -0.7$, $w_a = 0.5$

$w_0 = -1.3$, $w_a = 0.5$

$w = -0.7$, $w_a = -1.5$

$w = -1.3$, $w_a = -1.5$

$w = -1.3$

EdS

Open
Including $\delta_c$

Including $\Delta v$

Including c(M) hysteresis
Summary

• Matter 'power spectrum response' accurate to 1-2% for \( k < 5h \) Mpc\(^{-1}\) (Euclid or LSST lensing accuracy)

• No fitted parameters

• Publication:

• Code available:
  - https://github.com/alexander-mead/collapse

• Papers also contain accurate fitting functions for \( \delta_c \) and \( \Delta_v \) for a wide range of dark energy models